IDENTIFICATION AND PREDICTION OF THE DYNAMIC PROPERTIES OF RESISTANCE TEMPERATURE SENSORS

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ABSTRACT

The plunge test method and the self-heating test method represent two experimental techniques for identifying the dynamic properties of temperature sensors. The dynamic behaviour of a resistance temperature sensor can be described using transfer functions, which differ for the two test methods. It is possible to predict the sensor’s dynamic properties for the plunge test with a proper transformation of the identified model for the self-heating test. The main contribution of the presented research work is the software, based on virtual instrumentation, developed to identify and predict the dynamic properties of resistance temperature sensors. The excitation signal and the sensor’s response are utilized to identify its transfer function. The number of parameters for the approximation model is determined as a result of an optimization problem. The software was validated and then applied to identify and predict the dynamic properties of a commercial-grade Pt100 sensor. In this case study, the plunge test and the self-heating test were performed with a step change of the surrounding temperature and the supplied electrical power, respectively, under laboratory conditions. The relative difference between the predicted and the identified sensor’s time constants for the plunge test equals -7.4%, which is within the acceptance interval of ±10%. The tested resistance temperature sensor was therefore experimentally validated as being suitable for dynamic testing using the self-heating method.

Keywords: resistance temperature sensor; plunge test method; self-heating test method; identification of dynamic properties; prediction of dynamic properties; optimization problem
Highlights:

- Software was developed to identify and predict the resistance temperature sensors’ dynamic properties.
- An optimization problem is solved to determine the optimum transfer function of the sensor.
- A Pt100 sensor was tested using the plunge test and the self-heating test methods.
- The sensor’s dynamic properties for the plunge test were successfully predicted.

1 INTRODUCTION

A measurement of the process temperature is required in many industrial applications. Temperature sensors, e.g., resistance temperature sensors in nuclear power plants, are important elements of temperature measurement, control and safety systems. For this reason they must be accurate and have a good dynamic performance [1–5].

The dynamic properties of a temperature sensor may be identified theoretically, experimentally or by using a combined experimental-theoretical method [6–9]. The disadvantage of a solely theoretical approach is the demand for an exact knowledge of the sensor’s geometry, the properties of the contained materials and the working conditions [4]. This means that it is reasonable to define the structure of the model theoretically, while its parameters are identified by employing an appropriate experimental test method.

Reviews of the experimental methods for testing the dynamics of temperature sensors are presented in [10,11]. A resistance temperature sensor’s dynamic properties can be identified by utilizing the plunge test method and the self-heating test method [2]. The plunge test is performed by exposing the sensor to an external step change in the temperature of the surrounding fluid, e.g., air. The heat is then transferred through the sensor’s internal structure to the sensing element (or in the opposite direction if the step change occurs from a higher to a lower temperature). Under the self-heating test, the sensor is excited by an internal step change in the heat generation rate in the sensing element, resulting from an increased electrical current passing through it (Joule heating). The temperature of the sensing element rises and the heat is transferred through the sensor’s layers to the surrounding fluid. The self-
heating test method with a step change excitation is also known as the loop current step response (LCSR) test method [2,3].

The possibility for in situ testing is an advantage of the self-heating test method, since the process and installation conditions have a large effect on the dynamic properties of temperature sensors. In contrast, the plunge test has to be performed in a laboratory environment, where the process and installation conditions cannot always be reproduced. Thus the results have to be extrapolated to service conditions, which may lead to significant errors [3]. The suitability of the temperature sensor for the prediction of the dynamic properties on the basis of the self-heating test data can be validated by employing both test methods.

The aim of the presented research work was to assemble the measurement electronics and to develop the software for the (in situ) identification and prediction of the dynamic properties of resistance temperature sensors. The measurement electronics provide the measurement of temperature and the application of the internal excitation of a sensor during the self-heating test. The software is based on virtual instrumentation and was developed in the LabVIEW programming environment. Properly prepared excitation and response signals of the sensor are employed to estimate its transfer function. The number of approximation model parameters is determined as the solution of the optimization problem, which is an added value of the software. The significance of the selection of the optimum order of the approximation model is shown, e.g., in [12]. The software was validated on theoretical simulation cases.

A case study was carried out on a commercial-grade Pt100 sensor. The measurement system consisting of the sensor and the measurement electronics was calibrated. The tested sensor was then inserted into the air flow test rig, where the plunge test and the self-heating test were conducted under similar conditions. The developed software was employed to identify and predict the transfer functions, the unit step responses and the time constants of the sensor. The tested sensor was validated in terms of being suitable for the prediction of its dynamic properties for the plunge test on the basis of a properly transformed transfer function for the self-heating test. Some initial research work in the area of the experimental identification and prediction of the dynamic properties of resistance temperature sensors was
previously presented by the authors of this paper in a diploma thesis [13] and a conference paper [14].

This paper is structured as follows. The theoretical background of the dynamic properties of resistance temperature sensors is given in Section 2. In Section 3 an emphasis is put on the developed software and the measurement electronics for the (in situ) identification and prediction of the resistance temperature sensors’ dynamic properties. The case study is presented in Section 4.

2 THEORETICAL BACKGROUND

A multi-layer resistance temperature sensor typically comprises a sensing element on a support structure, an insulation material, a sheath and the lead wires. The dynamic properties of the sensor depend on its internal structure (especially at the sensing tip), the properties of the used materials and the process conditions, along with the thermodynamic and the transport properties of the surrounding fluid. The installation conditions, e.g., mounting into a thermowell, and ageing effects are also influential.

Different modelling approaches can be employed to determine the dynamic properties of a multi-layer resistance temperature sensor. Its step response can be approximated with the exact solution for the step response of an infinitely long, homogeneous cylinder [15]. In another approach, the system of partial differential equations governing the temperature field within the multi-layer sensor can be (approximately) solved using the finite-element method [16]. An advantage of this method is the possibility to find a solution for the complex geometries of temperature sensors [5,17].

The model in the form of a transfer function with a defined structure and unknown values of the parameters is suitable for the system identification. The transfer function of the multi-layer sensor can be derived by employing the lumped parameter model with the following simplifications and limitations taken into account. The computational domain of the sensor is discretized into as many cylindrical and concentric layers (a radial symmetry is taken into account) as necessary to achieve the required accuracy of the results. The spatial temperature variations within each layer are neglected. One-dimensional heat transfer in the
radial direction, constant properties of the used materials and a constant heat transfer coefficient between the sensor’s surface and the surrounding fluid are assumed [15,18]. The assumption of constant properties (linear model) is reasonable if the range of the temperature variations is narrow, e.g., up to 30 °C in air [19]. The heat balance equation is written in a finite-difference form for each layer. The Laplace transform is applied to a set of ordinary differential equations and the final solution is obtained in the form of a transfer function [18]. A steady-state and uniformly distributed temperature within the sensor is assumed before the application of the excitation. A small amount of heat is always being generated in a real resistance temperature sensor as a result of the electric current passing through the sensing element. Its influence on the initial temperature distribution within the sensor can be neglected [4].

During the plunge test, the resistance temperature sensor is excited externally by a step change in the fluid temperature; while during the self-heating test, it is excited internally by a step change in the heat generation rate. These two different input functions lead to different transfer functions for the two test methods. The transfer function for the plunge test, \( G_p(s) \), is a quotient between the Laplace transforms of the measured temperature, \( T(s) \), and the temperature of the surrounding fluid, \( T_f(s) \), while the transfer function for the self-heating test, \( G_{SH}(s) \), represents a quotient between the Laplace transforms of the measured temperature, \( T(s) \), and the heat generation rate in the sensing element, \( \dot{q} \) [4,18]. The transfer functions for zero initial conditions are given by [15]:

\[
G_p(s) = \frac{T(s)}{T_f(s)} = \frac{\prod_{j=1}^{m-1}(1 + \tau_{z,j}s)}{\prod_{j=1}^{n}(1 + \tau_{p,j}s)},
\]

\[
G_{SH}(s) = \frac{T(s)}{\dot{q}} = \frac{\prod_{j=1}^{m-1}(1 + \tau_{z,j}s)\prod_{j=m}^{n-1}(1 + \tau_{z,j}^{SH}s)}{\prod_{j=1}^{n}(1 + \tau_{p,j}s)},
\]

where \( \tau_{z,1}, \tau_{z,2}, \ldots, \tau_{z,m-1}, \tau_{z,m}^{SH}, \ldots, \tau_{z,n}^{SH} \) and \( \tau_{p,1}, \tau_{p,2}, \ldots, \tau_{p,n} \) are the modal time constants in the numerators and the denominators, respectively, \( m \) is the index of the layer that represents the
sensing element (e.g., \( m = 1 \) corresponds to the layer in the centre of the sensor), \( n \) is the number of all layers and \( K_{SH} \) is the static gain for the self-heating test. The static gain for the plunge test is equal to unity [15].

The modal time constants introduced with the self-heating test method are marked with the “SH” superscript. The values of the modal time constants depend on the heat capacities and the thermal resistances of the individual layers and the thermal resistance due to the convective heat transfer from the sensor’s surface. Each modal time constant is a negative inverse value of the corresponding zero or pole of the transfer function. The order of the denominators in the sensor’s transfer functions given by Eqs. (1) and (2) is equal to the selected number of layers, \( n \). For the plunge test, the order of the numerator depends on the relative location of the sensing element compared to the other layers, \( 0 \leq m - 1 \leq n - 1 \), while for the self-heating test it is equal to \( n - 1 \) [2,15].

The normalized unit step response resulting from the transfer function with \( m - 1 \) and \( n \) modal time constants in the numerator and in the denominator, respectively, is [15]:

\[
\Theta(t) = 1 - \sum_{i=1}^{n} a_i \exp\left(-t/\tau_{p,i}\right),
\]

(3)

where \( a_i \) is the general modal coefficient:

\[
a_i = \tau_{p,i}^{-m} \prod_{j=1}^{m-1} \left(\tau_{p,j} - \tau_{z,j}\right) \prod_{k=1}^{n} \left(\tau_{p,j} - \tau_{p,k}\right). \tag{4}
\]

The time constant is defined as the time required for the sensor’s response to reach 63.21% of its final steady-state value following a step change input [2,3].

The sets of the modal time constants in the denominators of the transfer functions are the same for both tests [15]. This means that it is possible to predict the transfer function for the plunge test on the basis of the experimentally identified transfer function for the self-heating test. The prediction is realized in the form of the transformed transfer function, \( G_{P,SH}(s) \), that is obtained by setting both the numerator term and the static gain in \( G_{SH}(s) \) to
The prediction accuracy of $G_{t\text{-SH}}(s)$ is limited by the unknown modal time constants in the numerator of $G_P(s)$. Some $(m - 1)$ modal time constants in the numerator of $G_{SH}(s)$ equal the modal time constants in the numerator of $G_P(s)$, while others $(n - m)$ are introduced with the self-heating test method. It is not possible to classify them, i.e., to recognize which ones are introduced with the self-heating test, if the transfer function $G_{SH}(s)$ is identified experimentally. However, if the thermal capacity between the sensing element and the central axis is negligible or the sensing element is located centrally inside the sensor, the transfer function for the plunge test, $G_P(s)$, does not contain any zeros at all [3,4].

The presented assumptions that have to be fulfilled in order to accurately predict the dynamic properties for the plunge test are met only to some degree by a real sensor. An example is the heat rate in the axial direction, which is ignored in the derivation of the presented models, but introduces additional zeros in the transfer function for the plunge test [20]. However, it is also stated in [20] that the deviations from the assumed one-dimensional heat transfer are often not significant in typical industrial resistance temperature sensors and thermocouples. The transformed transfer function, $G_{t\text{-SH}}(s)$, is therefore not expected to predict the identified $G_P(s)$ perfectly. The suitability of a real sensor for the prediction is estimated using the following criterion [3]:

$$\frac{\tau_{t\text{-SH}} - \tau_P}{\tau_P} \leq 10\%,$$

where $\tau_{t\text{-SH}}$ is the predicted time constant and $\tau_P$ is the identified time constant for the plunge test.

3 IDENTIFICATION AND PREDICTION OF THE DYNAMIC PROPERTIES

3.1 SOFTWARE ALGORITHM AND ITS VALIDATION

The computer program for the identification and prediction of the dynamic properties of the resistance temperature sensors was developed in the LabVIEW programming environment (National Instruments, Ver. 10.0). A block diagram of the software algorithm is presented in Fig. 1. The transfer function is estimated by employing the virtual instrument “SI
Estimate Transfer Function Model” contained in the LabVIEW System Identification Toolkit [21]. Properly prepared excitation and response signals are used to estimate the transfer function. The sampling rate of the signals has to be provided and the orders of the numerator and the denominator of the transfer function have to be selected as well. The form of the excitation signal can be arbitrary and not necessarily ideal, e.g., an ideal step input.

**Fig. 1:** Block diagram of the software algorithm for the identification and prediction of the dynamic properties of resistance temperature sensors.

The virtual instrument contains a multi-stage algorithm for the identification of the transfer function [22]. In the first step, the parameters of the ARX model (autoregressive model with external input) are estimated consecutively by employing the least squares method and the instrumental variable method. The solution is rearranged into the OE model (output error model) and optimized using the Gauss-Newton method. The obtained parameters are used to form the discrete transfer function, which is then converted into the continuous transfer function and finally refined using the Gauss-Newton method. The option to exclude the instrumental variable method was further implemented in the algorithm. The identification procedure without the instrumental variable method was found to be more stable for the majority of the discussed simulation and experimental cases with a step change input. Therefore, such an identification procedure was employed in all the cases presented in this paper.
The computational domain of the sensor has to be discretized into enough layers (nodes) in order to obtain an accurate lumped parameter model. On the other hand, both the internal structure and the order of the tested sensor are often unknown before the system identification. This limitation is avoided with an optimization algorithm that was developed to find the most appropriate transfer function in terms of its order. Approximation models of resistance temperature sensors with orders higher than four are not recommended [1]. Therefore, ten transfer functions with denominators from the 1st to the 4th order and numerators from the 0th to the 3rd order ($m \leq n$ is taken into account; see Eqs. (1) and (2)) are estimated based on the measured and pre-processed excitation and response signals. The corresponding approximation responses to the measured excitation signal are then calculated. The quality of each approximation response is evaluated using the standard error of estimate:

$$SEE(\Theta_s, \Theta_a) = \frac{1}{N_s - M} \sum_{i=1}^{N_s} ([\Theta_s(t_i) - \Theta_a(t_i)]^2),$$

where $\Theta_s(t_i)$ is the sensor’s response at the discrete time $t_i$ and $\Theta_a(t_i)$ is its approximation, $N_s$ is the number of samples in the signal and $M = m + n$ is the number of parameters in the approximation model. The transfer function with the corresponding approximation response that exhibits the minimum value of $SEE$ is selected as the most suitable transfer function for the sensor under test. The normalized unit step response and the time constant are then evaluated. If the prediction of the sensor’s dynamic properties for the plunge test is required, the optimum transfer function for the self-heating test is transformed by setting both the term in the numerator and the static gain to unity.

The algorithm for the identification of the optimum transfer function was validated using a known transfer function with the following parameters: $K = 1$, $\tau_{z,1} = 10$ s, $\tau_{p,1} = 15$ s, and $\tau_{p,2} = 5$ s. The unit step input and the calculated response of this dynamic system were used to estimate the original transfer function. Gaussian white noise with a selected standard deviation of its probability density function, $s_n$, was added to the excitation and response signals (a different noise pattern for each signal), which have a sampling frequency of 10 Hz and a length of 60 s. The estimated parameters of the transfer functions for different values of the standard deviation $s_n$ are presented in Table 1. If the signals were noise-free, the estimated parameters were identified accurately, while added noise negatively influenced the accuracy...
of the estimated parameters. In an example with $s_n = 0.01$, two additional modal time constants in the denominator of the estimated transfer function appeared: $\tau_{p,3,4} = (0.00025 \pm 0.043i)$ s. These probably result from overfitting the noisy data. From the theoretical point of view any pair of zeros or poles of the resistance temperature sensors’ transfer functions is not expected to have the form of complex conjugates.

Table 1: Estimated parameters.

<table>
<thead>
<tr>
<th>$s_n$ [-]</th>
<th>$K$ [-]</th>
<th>$\tau_{z,1}$ [s]</th>
<th>$\tau_{p,1}$ [s]</th>
<th>$\tau_{p,2}$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>10</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1.000</td>
<td>10.020</td>
<td>15.025</td>
<td>5.000</td>
</tr>
<tr>
<td>$2.5 \times 10^{-3}$</td>
<td>0.999</td>
<td>9.825</td>
<td>14.816</td>
<td>4.967</td>
</tr>
<tr>
<td>$5 \times 10^{-3}$</td>
<td>0.999</td>
<td>9.654</td>
<td>14.726</td>
<td>4.881</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>0.997</td>
<td>9.665</td>
<td>14.414</td>
<td>5.059</td>
</tr>
</tbody>
</table>

3.2 MEASUREMENT ELECTRONICS

The measurement electronics were assembled in order to measure the temperature and to perform the internal excitation of the sensor during the self-heating test. A scheme of the measurement electronics is presented in Fig. 2. The resistance temperature sensor is connected to a Wheatstone bridge along with three thermally stable resistors with a nominal resistance $R_0 = 120 \, \Omega$ and an accuracy class of 0.1%. A DAQ board (National Instruments, USB-6341) is used to generate and measure the bridge’s voltage supply, $U_{ref}^*$ and $U_{ref}$, respectively, as well as to measure the bridge’s voltage output, $U_o$, and the voltage drop over the thermally stable resistor with resistance $R_0^* = 120.035 \, \Omega$, $U_{ref}^*$. In case of the self-heating test the supply voltage is amplified using an electrical amplifier. The electrical resistance of the temperature sensor is calculated using the equation for a constant-voltage Wheatstone bridge with three non-active resistors having the nominal resistance $R_0$:

$$R_s = \gamma R_0 \frac{1 - 2U_o / U_{ref}}{1 + 2U_o / U_{ref}},$$ (7)
where $\gamma$ is a correction factor that was introduced because the actual resistances of the non-active resistors vary within a specified resistance accuracy of 0.1% and due to other possible systematic errors in the temperature measurement system. The correction factor is determined by calibration. The electrical resistance of the sensor is used to calculate the temperature by employing the relationship between resistance and temperature from the standard IEC 60751 [23]:

$$R_s(T) = R_{s0} \left(1 + AT + BT^2\right),$$  

(8)

where $A = 3.9083 \times 10^{-3} \, ^\circ C^{-1}$ and $B = -5.775 \times 10^{-7} \, ^\circ C^{-2}$ are constants and $R_{s0} = 100 \, \Omega$ is the nominal resistance at 0 $^\circ C$ for the Pt100 temperature sensor. The electrical current passing through the sensing element is determined from the equation $I_s = U_{ref} / R_s$ and the supplied electrical power is calculated as $P = R_s I_s^2$. The input electrical power is converted to the generated heat rate in the sensing element, so it is taken into account as the actual excitation signal during the self-heating test.

![Fig. 2: Scheme of the measurement system.](image-url)
4 DYNAMIC PROPERTIES OF A Pt100 SENSOR: A CASE STUDY

4.1 MEASUREMENT SYSTEM

The measurement system presented in Fig. 2 was employed to carry out a case study. The tested, commercial-grade Pt100 resistance temperature sensor (TetraTec Instruments, WIT-S) is shown in Fig. 3(a). The external diameter of the sensor’s sheath is 3 mm. Four layers of different materials, the concentric internal structure and the radially located sensing element within the sensor assembly, are evident from an X-ray image of its sensing tip, presented in Fig. 3(b). A small deviation from ideal concentricity and radiality is also evident.

![Fig. 3: Pt100 resistance temperature sensor under test: (a) external appearance and (b) X-ray image of the sensor’s sensing tip.](image)

The tested sensor was connected to the measurement electronics. This measurement system was calibrated in the temperature range from 15 °C to 65 °C with the reference temperature having an expanded measurement uncertainty of 0.02 °C. The static calibration was conducted in the Laboratory of Measurements in Process Engineering at the Faculty of Mechanical Engineering, University of Ljubljana (ISO 17025 accredited laboratory).

The air flow test rig was used to conduct the plunge tests. It has two perpendicularly positioned air channels and a switching channel element at the junction of the channels [7]. An air flow with a set velocity and temperature can be maintained independently in each channel with the help of a radial fan, an electric heater and a flow conditioner. The temperature sensor under test is inserted into the switching channel element that can be
rapidly moved between its two operating positions by means of a pneumatic cylinder. The position of the channel element is determined through the voltage output from the potentiometer, $U_p$. Throughout the test, the sensor is kept fixed; it does not move together with the channel element. Consequently, the air stream from the other channel is directed to flow around the sensor when the channel element’s position is changed and so a step change in the temperature is generated. The new steady temperature following the step change is established in less than 0.1 s [7]. The self-heating tests were also conducted in the air flow test rig. During the self-heating test the air stream from the selected channel flows around the sensor and provides stable temperature and velocity conditions. The internal excitation comes from the measurement electronics.

The signal acquisition and processing were realized in particular LabVIEW-based program. The prepared signals were then analysed with the developed software for the identification and prediction of the dynamic properties of the resistance temperature sensors (see Section 3.1).

4.2 TEST PROCEDURE

During the plunge test the sensor was excited by a switch from an air stream with a temperature of 50 °C to an air stream with a temperature of 30 °C. The step change was realized from the higher to the lower temperature due to more stable air flow at lower temperatures. The air velocity was set to 2 m/s in each channel. Since the sensor’s sensing tip was located at approximately half of the inner height of the channel element, we assumed that the local switch between the air streams happened at the moment when the channel element passed half of the distance between its two operating positions. The temperature excitation signal was formed as an ideal step change realized at the time when the voltage output signal from the potentiometer (that is proportional to the position of the switching channel element) reached 50% of its normalized final value. The electrical current passing through the sensor was set to 1 mA.

During the self-heating test the sensor was excited by a step change in the supplied electrical power from 0.11 mW to 100 mW. This step change was technically realized as a
step change in the supply voltage of the Wheatstone bridge. The initial electrical current passing through the sensor was 1 mA, which was then increased to approximately 28.8 mA. The sensor was immersed in an air stream with 30 °C and 2 m/s during the self-heating test. A final steady-state temperature in the sensing element of approximately 53.5 °C was established.

Before the start of each test and the acquisition of the signals, the sensor was left in the air flow for more than 350 s in order to ensure the steady-state initial condition. The recommended supply current of 1 mA was not expected to result in a significant temperature gradient within the sensor due to the self-heating and thus a uniformly distributed temperature was assumed.

A sampling rate of 1 kHz was used to record the data. A moving average with a sample length of 0.02 s was applied to the preliminarily normalized signals in order to eliminate the noise with a frequency of 50 Hz. For each test method, ten repetitions of the measurements were performed. The obtained signals were then ensemble averaged. The original signals for both test methods had lengths of 250 s, with 50 s before the step change.

4.3 RESULTS AND DISCUSSION

For the purposes of the system identification, the lengths of the signals before the step change were set to 1 s and the total lengths were varied in order to analyse its influence on the prediction error, $(\tau_{t,SH} - \tau_P) / \tau_P$. Fig. 4 shows the prediction error as a function of the signal length for the self-heating test, $t_{\text{sig,SH}}$, while the signal length for the plunge test, $t_{\text{sig,P}}$, was set to 102 s. The prediction error decreases (in terms of the absolute value) as the signal length increases. It is reasonable to select the signal length from the region where the prediction error becomes nearly constant, e.g., $t_{\text{sig,SH}} = 90$ s in our case.
Fig. 4: Prediction accuracy as a function of the signal length for the self-heating test.

The values of the SEE of the approximation responses resulting from all the estimated transfer functions for the selected signal lengths of $t_{\text{sig},p} = 102$ s and $t_{\text{sig},SH} = 90$ s are presented in Fig. 5. The minimum values of the SEE are equal to $1.63 \times 10^{-3}$ and $1.92 \times 10^{-3}$ for the plunge test and for the self-heating test, respectively. The following transfer functions were identified as the most suitable for the sensor under test:

$$G_p(s) = 1.004 \frac{1}{(1+15.188s)(1+3.056s)},$$

$$G_{SH}(s) = 0.999 \frac{(1+9.660s)}{(1+13.775s)(1+3.072s)}.$$  \hspace{1cm} (9) \hspace{1cm} (10)

From the theoretical point of view, the structures of the identified transfer functions are obtained if the insulation material and the sheath are considered as one layer and if the heat capacity between the sensing element and the central axis of the sensor is neglected. In [11] it is stated that an equivalent transfer function of the 2nd order can be attributed to the majority
of industrial resistance temperature sensors. The pre-processed signals and the approximation responses for both test methods are shown in Figs. 6 and 7.

**Fig. 5:** Standard error of estimate of the approximation responses for different orders of the model.
Fig. 6: Pre-processed signals and the approximation response for the plunge test.

Fig. 7: Pre-processed signals and the approximation response for the self-heating test.
The transfer function for the plunge test was predicted by the following transformed transfer function:

\[ G_{t-SH}(s) = \frac{1}{(1+13.775s)(1+3.072s)}. \]  \hfill (11)

The unit step responses for all three models are shown in Fig. 8. The values of the time constants are: \( \tau_{SH} = 5.36 \text{s} \), \( \tau_{t-SH} = 17.21 \text{s} \) and \( \tau_p = 18.58 \text{s} \). The prediction error, \( (\tau_{t-SH} - \tau_p) / \tau_p \), is equal to –7.4% and, as a result, it lies within the specified acceptance interval of ±10%. The sensor under test was therefore experimentally validated to be suitable for predicting its dynamic properties for the plunge test based on properly transformed self-heating test data.

![Normalized unit step responses of the Pt100 resistance temperature sensor under test.](image)

**Fig. 8:** Normalized unit step responses of the Pt100 resistance temperature sensor under test.
Possible reasons for the difference in the time constants have to be addressed. Linear models of a resistance temperature sensor were taken into account, since the temperature changes were 20 °C and 23.5 °C (less than 30 °C; see Section 2) during the plunge test and during the self-heating test, respectively. However, nonlinearity could be introduced due to the different directions of the step perturbations applied to the tested sensor, i.e., it was cooled down during the plunge test and heated up during the self-heating test. A simulation of this effect on thermocouple’s step response is shown in [5]. As a shape simplification, the non-ideal nature of tested sensor’s internal structure (see Section 4.1) was not taken into account.

The accuracy of the estimated transfer function depends on a variety of parameters, such as the signal length, the sampling frequency and the signal noise. A careful visual examination of the resulting approximation response is recommended. In the case of inappropriate results the identification procedure should be repeated, using, e.g., signals with a modified length or down-sampled. It is also reasonable to minimize the noise of each signal, e.g., by employing a moving average or an ensemble average of a few measured signals. The system identification procedure has the option to exclude the instrumental variable method (see Section 3.1). Another possibility is to change the convergence criteria of the Gauss-Newton algorithms within the system identification process. Further testing with different settings and by employing both the simulation and the experimental signals should be performed in order to analyse and improve the capabilities of the developed software. The validation signals should be obtained from dynamic systems with different dynamic properties, which are excited with input signals of different types.

5 CONCLUSIONS

The software for the identification and prediction of the dynamic properties of resistance temperature sensors was developed on the basis of virtual instrumentation. The capabilities of the algorithms implemented in the LabVIEW System Identification Toolkit make it possible to identify the sensor’s transfer function by utilizing a stimulation signal and the sensor’s corresponding response. The order of the approximation model is not predefined, but it is determined by an optimization algorithm using the selected objective function (SEE), which is the added value of the developed software. The software was validated on the
selected dynamic system. A step input and the system’s response signals (additional noise was added to both) were used to estimate the original transfer function. The accuracy of the identified parameters was negatively affected by the added noise. The results obtained with the developed software have to be visually examined, since the accuracy of the estimated transfer function is influenced by parameters such as the signal length, the sampling frequency, the signal noise and also by some settings of the system identification procedure, which to some extent limit the applicability of the computer program. Additional effort will have to be put into the further development of a more robust program.

The measurement electronics consisting of an electrical circuit with a Wheatstone bridge, a DAQ board and a voltage amplifier were assembled with the purpose of measuring the temperature with a Pt100 resistance temperature sensor and conducting a self-heating test. A commercial-grade Pt100 sensor was connected to the measurement electronics. This measurement system was calibrated. Self-heating tests and plunge tests with step input signals were performed on the sensor under similar conditions in a laboratory environment. The dynamic properties of the sensor were identified, predicted and analysed with the developed software. The most suitable transfer function for the plunge test method has two poles, while the most suitable transfer function for the self-heating test has two poles and one zero. The latter transfer function was transformed by setting both the term in the numerator and the static gain to unity, with the purpose of predicting the transfer function for the plunge test. The suitability of the sensor for this prediction was experimentally validated, since the relative difference between the corresponding time constants was -7.4%, which is within the acceptance interval of ±10%. The assumptions regarding the internal structure and the properties of the materials in the sensor’s sensing tip and the dominant heat transfer in the radial direction between the sensing element and the surroundings of the sensor are therefore met sufficiently well. For the purpose of this paper, the temperature sensor under test was perturbed with step change excitation signals. It is also possible to conduct both tests with perturbation signals of different forms, which represents an option for further research work.

The findings from this research work could be helpful in terms of the further development of measurement electronics and software for the (in situ) identification and prediction of temperature sensors’ dynamic properties and also the development of resistance temperature sensors with an improved prediction accuracy.
5 REFERENCES


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