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Dynamic pressure corrections in a clearance-sealed piston prover for gas flow measurements

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Abstract
The dynamic pressure effects and their corrections in a high-speed, clearance-sealed realization of a piston prover for gas flow measurements are discussed. The experimental results show the deterministic, rather than stochastic, nature of the dynamic pressure conditions and, consequently, the repeatable nature of their influence on the flow measurements. The experimental validation proves the advantage of the polytropic/adiabatic pressure correction model, which was proposed by the authors, as compared with the isothermal pressure correction model. The paper ends with an estimation of the measurement uncertainty related to the pressure corrections using either the adiabatic or isothermal model.

1. Introduction
The piston-prover concept is widely used for primary standards in the field of gas flow measurements [1–7]. The general principle of operation is based on determining the time interval that a piston needs to pass a known volume of gas at a defined pressure and temperature.

The experimental work of this paper was performed on a commercially available, clearance-sealed realization of the piston prover [8–10]. The piston is made of a graphite composite and the cylinder is made of borosilicate glass. The piston and the cylinder are closely fitted, with a clearance of the order of 10 µm. The passage of the piston is detected by infrared light emitters and sensors. The base of the piston prover holds the timing crystal, the barometric pressure sensor and the computer. The gas gauge pressure is measured by a fast-response pressure sensor, which is connected to the flow system at the entrance to the cylinder employing the pressure impulse line. The gas temperature is measured at the same location using a thermistor temperature sensor. Due to its relatively high heat capacity (a probe diameter of about 1 mm to 2 mm), the measured temperatures represent nearly time-averaged values during the high-speed operation of the piston prover.

The piston prover under discussion employs the following measurement model for the volume flow rate at the barometric pressure $P_a$ and the gas temperature $T$:

$$q_v(P_a, T) = \left( \frac{V_m}{\Delta t} + q_{v,1} \right) \varepsilon_p,$$

where $V_m$ is the measuring volume of the gas collected by the piston prover during the timing cycle $\Delta t = t_2 - t_1$, $q_{v,1}$ is the clearance leakage flow (more precisely, its Poiseuille component, whereas its Couette component is considered as the reduced effective diameter of the cylinder in $V_m$) and $\varepsilon_p$ is the pressure correction factor, which takes into account the deviations of the actual pressure conditions from the barometric pressure. The equation for $\varepsilon_p$, which is originally employed in the piston prover and termed the isothermal pressure correction in this paper, has the form

$$\varepsilon_p = 1 + \frac{P_2}{P_a} + \frac{P_2 - P_1}{P_a} \frac{V_d}{V_m},$$

where $P_1$ and $P_2$ are the gauge pressures at times $t_1$ and $t_2$, respectively, and $V_d$ is the connecting volume of the gas between the selected inlet transfer point and the piston at time $t_1$. 
The derivation from the law of conservation of mass points out that the validity of the pressure correction model in equation (2) depends on the following assumptions [11]:

(i) ideal gas: this is valid due to the relatively small pressure changes from the barometric pressure and so the gas compressibility effects can be considered as negligible;
(ii) spatially homogeneous pressure changes: this is at least valid for the compact internal volume of the piston prover, the linear dimensions of which are small compared with the wavelengths of the pressure oscillations;
(iii) isothermal system: this is only valid if the dynamic processes in the gas are slow with respect to the heat exchange.

Because the pressure oscillations in the piston prover under discussion are of the order of a few tens of hertz, the validity of the isothermal assumption is questionable; fast processes with respect to heat exchange can often be considered as polytropic or quasi-adiabatic. In [11] the authors of this paper derived and theoretically validated an equation for polytropic/adiabatic pressure correction, which can be written as

\[ \varepsilon_p = 1 + \frac{\bar{p}_{12}}{p_a} + \frac{1}{\gamma} \left( \frac{p_2 - \bar{p}_{12}}{p_a} + \frac{p_2 - p_1}{p_a} \cdot \frac{V_d}{V_m} \right), \]  

(3)

where \( \bar{p}_{12} \) is the time-averaged value of the gauge pressure during the timing cycle and \( \gamma \) is the polytropic index. In the limit of adiabaticity, \( \gamma \) is the ratio of the specific heats or the adiabatic index. For dry air the adiabatic index is approximately \( \gamma = 1.4 \). If \( \gamma = 1 \) is considered, the polytropic/adiabatic measurement model (3) transforms into the isothermal measurement model (2).

The aim of this paper is a metrological analysis of the pressure correction factors in the piston prover. The main contribution is an experimental validation of the proposed polytropic/adiabatic measurement model for pressure corrections and the corresponding decrease in its contribution to the uncertainty of the gas flow measurements. Section 2 describes the measurement system, which is then used for measuring the pressure variations at different flow rates. These pressure variations are discussed in terms of their deterministic or stochastic dynamic nature. Section 3 deals with the properties of the pressure correction factors. The experimental validation test proves the advantage of the polytropic/adiabatic model in comparison with the isothermal model. Section 4 presents an analysis of the measurement uncertainty of the pressure correction factors.

2. Pressure characteristics

Measurements of the pressure characteristics of the piston prover (Sierra Instruments, Cal=Trak SL-800-44, measuring range 0.5 sl min\(^{-1}\) to 50 sl min\(^{-1}\)) were carried out for different flow rates within its measuring range using clean, oil-free, dry air. The measurement system is schematically presented in figure 1. The stable mass flow rate through the piston prover was set with the help of a pressure regulator and a selected critical nozzle (TetraTec Instruments, array of five Venturi-shaped critical nozzles) operating under sonic conditions.

All the flow rate values given in this paper represent flow measurements by the piston prover calculated as the standard volume flow rate at \( P_a = 101.325 \) kPa and \( T_a = 293.15 \) K:

\[ q_1 = \frac{\rho(P_a, T)}{\rho(P_s, T_s)} q_s(P_s, T_s), \]  

(4)

where the REFPROP database [12] was used to determine the air densities \( \rho(P_a, T) \) and \( \rho(P_s, T_s) \). The flow measurements of the piston prover were controlled using an RS-232 serial communication with the ASCII protocol, which initiates the measurement and acquires the data query stream, including the uncorrected volume flow rate \( V_m/\Delta t \), the barometric pressure \( P_a \), the temperature \( T_a \) the gauge pressures \( p_1 \) and \( p_2 \), etc [13]. In order to study the pressure variations during the piston prover’s operation and to have the possibility to determine the time-averaged pressure \( \bar{p}_{12} \) we used an additional ‘external’ gauge pressure sensor (Validyne, P855, measuring range \( -1.4 \) kPa to 1.4 kPa, voltage output \(-5 \) V to 5 V, low pass filter at 250 Hz/–3 dB), which was connected in parallel with the internal gauge pressure sensor. Its voltage output was measured with a DAQ board (National Instruments, USB-6251 BNC). The processing of the measurement signals was realized with LabVIEW software (National Instruments, Ver. 10.0).

For each of the seven flow rates (at about 1.5 sl min\(^{-1}\), 2.7 sl min\(^{-1}\), 4.8 sl min\(^{-1}\), 8.6 sl min\(^{-1}\), 15 sl min\(^{-1}\), 27 sl min\(^{-1}\) and 46 sl min\(^{-1}\) ) 20 consecutive repetitions of the measurements with a simultaneous acquisition of the readings of the piston prover and the external pressure sensor were carried out. Figure 2 shows an example of the measured pressure response during one measurement cycle of the piston prover for a flow rate of 8.6 sl min\(^{-1}\). It schematically demonstrates the extraction of the timing cycle from this external pressure sensor’s signal. Although the length of the timing cycle \( \Delta t \) is known from the piston prover’s measurands, its position in time is undetermined (the output signals of the optical sensors, which are used in the piston prover’s electronics to trigger the time values \( t_1 \) and \( t_2 \), are not easily accessible to the user). The real-time estimation of the \( t_2 \) value was achieved by a software trigger, the level of which was set at approximately 80% of the average pressure change at the end of the measuring cycle, and by an additional subtraction of a time lag of about 0.015 s from the corresponding trigger time. Using the described procedure we achieved a suitable synchronization of the measured pressures \( p_1 \) and \( p_2 \) from the external and internal pressure signals, with differences of less than 10 Pa (it has to be emphasized that even a less accurate estimate of the timing cycle on the external pressure signal would not affect the estimate of its time-averaged value \( \bar{p}_{12} \), which will only be used in the polytropic/adiabatic model).

Figure 3 shows the extracted timing cycles of the pressure responses for two different flow rates of 8.6 sl min\(^{-1}\) and 27 sl min\(^{-1}\). Each graph consists of four selected measurements from 20 repetitions (5th, 10th, 15th and 20th series). It is evident that the pressure variations in the piston prover are not stochastic but deterministic, because the measured pressure signals practically overlap (see [11] for a
frequency analysis of the observed pressure oscillations and a discussion of the reasons for their occurrence).

The predominantly deterministic nature of the dynamic pressure conditions is, consequently, also shown in the values of the characteristic pressures: figure 4 shows the pressures $p_1$ and $p_2$ (internal pressure sensor) and the time-averaged pressures $\bar{p}_{12}$ (external pressure sensor) for all 20 repetitions of the measurements at a certain flow rate. The entire range of pressure oscillations in the timing cycle is illustrated by the values of the maximum and minimum pressures. The largest difference between $p_2$ and $p_1$, which approaches the span between the maximum and minimum pressures, occurs at a flow rate of 8.6 sl min$^{-1}$. As already reported in [11], this flow range represents the most intensive resonance effects of the piston oscillator, which are excited by the self-sustained oscillations of the inlet flow in the cylinder below the moving piston. At higher flow rates, $p_1$ remains smaller than $\bar{p}_{12}$ and $p_2$ remains almost equal to $\bar{p}_{12}$. Such repeatable pressure characteristics indicate a certain degree of synchronization between the pressure response and the piston movement along the cylinder (other positions of the optical sensors would change the values of $p_1$ and $p_2$).

3. Isothermal versus adiabatic pressure corrections

We assume that the presented dynamic conditions are fast enough to consider a quasi-adiabatic process. The isothermal and adiabatic pressure correction factors $\epsilon_p$ are calculated using equations (2) and (3), respectively. We take into account the measured pressure values from figure 4, the measured barometric pressures $P_a$ ($\approx 98.5$ kPa), the dimensionally calibrated measuring volume $V_m = 118.2$ ml, the estimated connecting volume $V_d = 200$ ml (in this value there is about 30 ml of the volume between the critical nozzles and the entrance to the piston prover, which will be considered as the source of the uncertainty; see section 4) and the polytropic index of dry air close to $\gamma = 1.4$ for the adiabatic model. The relative values of the pressure corrections $(\epsilon_p - 1)$ determined for both models are presented in figure 5. They range from about 0.15% at the smallest flow rate to about 1% at the largest flow rate. The dashed line in the graph shows the share of the influence of the time-averaged pressure $\bar{p}_{12}/P_a$. Figure 5
Dynamic pressure corrections in a clearance-sealed piston prover

Figure 3. Pressure responses during the timing cycle at two different flow rates (four repetitions).

(a) Flow rate of 8.6 sl/min.

(b) Flow rate of 27 sl/min.

Figure 4. Characteristic pressures in the timing cycle.

also presents the difference between the pressure correction factors for the isothermal and adiabatic models, which can be written as

$$\delta_{T-A} = \frac{\gamma - 1}{\gamma} \left( \frac{p_2 - \bar{p}_{12}}{P_a} + \frac{p_2 - p_1}{P_a} \frac{V_d}{V_m} \right).$$

Figure 5. Relative values of the pressure corrections.

The largest value of the difference $\delta_{T-A}$ is about 0.1% at a flow rate of 8.6 sl min$^{-1}$.

An important question that remained open after deriving and theoretically validating the polytropic/adiabatic model in [11] was the experimental confirmation of its validity. For this reason, the following validation experiment was prepared:

(i) the sonic nozzles were used to ensure a constant mass flow rate to the piston prover;

(ii) the outlet flow conditions were modified to influence the dynamic conditions in the piston prover and so the values of the dynamic pressure corrections;

(iii) if the pressure correction model of the piston prover is appropriate, it would take into account the variation in the pressure conditions and therefore the output of the mass flow calculation would not change after the modification.

The following results were obtained by the validation experiment at a flow rate of 8.6 sl min$^{-1}$. A modification of the dynamic conditions was made by a restriction at the outlet of the piston prover giving an additional pressure drop of about 0.5 kPa. The flow restriction was realized by connecting a tube with an inner diameter of 4 mm. The measured values of the characteristic pressures before the modification (left) and after the modification (right) are presented in figure 6(a).

In addition to the higher value of the average pressure, the flow restriction significantly reduced the influential dynamic components $p_2 - \bar{p}_{12}$ and $p_2 - p_1$, and thereby the magnitudes of the dynamic pressure corrections. The corresponding relative changes in the piston prover’s mass flow readings are shown in figure 6(b). While the modification of the dynamic pressure conditions caused a systematic change in the measured flow rate of about 0.06% when using the isothermal model, no notable systematic changes were observed in the case of the adiabatic model. This confirms the advantage of the adiabatic pressure correction model in comparison with the isothermal pressure correction model. The same conclusion regarding the adiabatic model was also found on the basis of other measurements at a flow rate of 8.6 sl min$^{-1}$ using different lengths of the outlet connecting tube (the gauge pressures in the piston prover were kept below the recommended maximum value of 1.25 kPa [14]).
values into the measurement uncertainty of the characteristic pressure correction factor by the piston prover is limited to the uncertainty of the pressure. In this paper, the uncertainty analysis of the flow rate measured corrections

4. Measurement uncertainty of the pressure

Similar validation experiments were also performed at higher flow rates. Although the corresponding dynamic conditions have similar frequencies and so the validity of the adiabatic model is expected, the properties of the piston prover under discussion did not enable us to attain representative validation results. The main problems were related to the smaller dynamic pressure corrections (smaller differences between the adiabatic and the isothermal models), reduced repeatability of the mass flow readings (mainly related to the time discretization effects that, for example, reach 0.1% at the largest flow rates) and the inability to generate sufficient modifications of the dynamic pressure conditions by an additional pressure drop at the outlet.

4. Measurement uncertainty of the pressure corrections

In this paper, the uncertainty analysis of the flow rate measured by the piston prover is limited to the uncertainty of the pressure correction factor \( \varepsilon_p \). The main contributions can be divided into the measurement uncertainty of the characteristic pressure values \( p_1 \) and \( p_2 \) and \( \bar{p}_{12} \), the uncertainty related to the connecting volume \( V_d \) and the uncertainty related to the polytropic index \( \gamma \). The contributions from the parameters \( P_a \) and \( V_m \)—both the relative standard uncertainties \( u(P_a)/P_a \) and \( u(V_m)/V_m \) are about 0.025%—can be neglected due to their minor influence on the uncertainty of the pressure correction (they have to be accounted for in the entire measurement uncertainty of the mass flow rate where they appear in the estimate of the gas density \( \rho(P_a,T) \) and the uncorrected volume flow rate \( V_m/\Delta t \), respectively). The measurement uncertainty evaluation presented here refers to each single realization of the 20 repetitive measurements at each flow rate and is performed in accordance with the GUM [15].

Considering the adiabatic equation (3) and \( u_x(\varepsilon_p) = (\partial s_p/\partial x_i)u(x_i) \), the contributions of the standard measurement uncertainty of the characteristic pressures can be written as

\[
\begin{align*}
    u_{p_1}(\varepsilon_p) &= \frac{1}{\gamma} \frac{V_d}{V_m} u(p_1), \\
    u_{p_2}(\varepsilon_p) &= \frac{1}{\gamma} \left( 1 + \frac{V_d}{V_m} \right) u(p_2), \\
    u_{\bar{p}_{12}}(\varepsilon_p) &= \frac{\gamma - 1}{\gamma} u(\bar{p}_{12}) = 2 \text{ Pa}.
\end{align*}
\]

The standard uncertainties of the pressure values \( p_1 \) and \( p_2 \) measured by the internal pressure sensor of the piston prover are estimated to be \( u(p_1) = u(p_2) = 5 \text{ Pa} \), and the standard uncertainty of the time-averaged pressure \( \bar{p}_{12} \) measured by the external pressure sensor is estimated to be \( u(\bar{p}_{12}) = 2 \text{ Pa} \). These pressure uncertainties include the contribution of the measurement errors and their uncertainties from the last calibration of the pressure sensors, the contribution of their time drift and, for the instantaneous pressure values \( p_1 \) and \( p_2 \), also the contribution of the dynamic errors of the pressure measurements. The internal pressure sensor of the piston prover is connected to a pressure tap at the entrance to the cylinder with a pressure impulse line of length about 220 mm and an inner diameter of about 2 mm. The natural frequency of such a fluid oscillator is about 350 Hz [16]. Taking into account that the largest amplitudes of the pressure oscillations are about 200 Pa and that the main frequency components fall in the frequency range up to 40 Hz [11], the maximum dynamic measurement error can be estimated as 2.5 Pa.

The combined uncertainty of the pressure correction factor caused by the uncertainties of all the characteristic pressure values is calculated as

\[
u_p(\varepsilon_p) = \sqrt{u_{p_1}^2(\varepsilon_p) + u_{p_2}^2(\varepsilon_p) + u_{\bar{p}_{12}}^2(\varepsilon_p)}, \tag{7}\] where the individual contributions are assumed to be uncorrelated. (This assumption may lead to some overestimation of the combined uncertainty. If a certain correlation of the pressure values \( p_1 \) and \( p_2 \), which are measured with the same pressure sensor, is assumed, the part of the pressure correction model including \( (p_2 - p_1) \) would have a smaller contribution to the combined uncertainty.)

The contribution of the standard uncertainty of the connecting volume to the standard uncertainty of the pressure

\[
\begin{align*}
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correction factor can be written as
\[ u_{V_d}(\varepsilon_p) = \frac{1}{\gamma} \frac{p_2 - p_1}{P_a} \frac{V_d}{V_m}. \] (8)

With the uncertainty of the connecting volume we want to encompass the uncertainty of the estimation of the connecting volume and the uncertainty of the model’s assumption of homogeneous pressure changes within the connecting volume. The estimate of \( V_d = 200 \) ml is considered to be the mid-point of a rectangular distribution with a half-width of 30 ml and therefore the standard uncertainty is \( u(V_d) = 30/\sqrt{3} \) ml. The volume of 30 ml corresponds approximately to the connecting volume outside the piston prover.

The contribution of the standard uncertainty of the polytropic index can be written as
\[ u_\gamma(\varepsilon_p) = -\frac{1}{\gamma} \left( \frac{p_2 - \bar{p}_2}{P_a} + \frac{p_2 - p_1}{P_a} \frac{V_d}{V_m} \right) u(\gamma). \] (9)

The systematic error \( \delta_{T-A} \) of the isothermal pressure correction is included in the measurement uncertainty using the SUMU\(_{\text{max}}\) method, where the absolute value of the systematic error is added to the value of the expanded measurement uncertainty [15, 17]. The equivalent combined standard measurement uncertainty of the isothermal pressure correction factor can therefore be estimated as
\[ u(\varepsilon_p) = \sqrt{u_\gamma^2(\varepsilon_p) + u_{V_d}^2(\varepsilon_p) + \delta_{T-A}^2}. \] (10)

The systematic error \( \delta_{T-A} \) of the isothermal pressure correction is included in the measurement uncertainty using the SUMU\(_{\text{max}}\) method, where the absolute value of the systematic error is added to the value of the expanded measurement uncertainty [15, 17]. The equivalent combined standard measurement uncertainty of the isothermal pressure correction factor can therefore be estimated as
\[ u(\varepsilon_p) = \sqrt{u_\gamma^2(\varepsilon_p) + u_{V_d}^2(\varepsilon_p) + |\delta_{T-A}|/2}, \] (11)

where the coverage factor \( k = 2 \) is assumed.

Figure 7 shows the values of the individual contributions and the combined standard uncertainty of the pressure correction factors for the isothermal and adiabatic models. In the case of the isothermal model (2), the combined standard uncertainty exceeds 0.07%. The largest contribution is represented by the systematic error, because the adiabatic nature of the dynamic pressure conditions is not considered. The use of the adiabatic model (3) reduces the largest combined standard uncertainty by more than three times, to about 0.023%.

5. Conclusions

The purpose of this paper was to present the dynamic pressure conditions in a high-speed, clearance-sealed realization of the piston prover and to analyse their influence on gas flow measurements. Here, we summarize some of the most important findings.

(i) The experimental results show that the dynamic pressure conditions are principally of a deterministic nature. Consequently, the uncorrected pressure dynamic effects are not reflected mainly as a reduced repeatability of the flow meter, but as systematic errors. The largest time-averaged gauge pressures during the piston prover’s operation reach values of 0.75 kPa and the largest amplitudes of the pressure oscillations are about 0.2 kPa.

(ii) The experimental validation proves the advantage of the polytropic/adiabatic model of the pressure correction factor by a comparison with the originally employed, isothermal model. The measurement uncertainty analysis of the pressure correction factor shows that its maximum contribution, expressed as the standard uncertainty, was reduced by nearly three times, from about 0.07% to 0.023%, when using adiabatic instead of isothermal corrections.

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In addition to the further optimization of the correction algorithms for the pressure effects in high-speed realizations of piston provers, one possible direction for their improvement would be a decrease in the pressure changes. Here it would be useful to work on reducing the amplitudes of the pressure oscillations (at least partially related to the resonance effects of the piston oscillator excited by the inlet flow instabilities) as well as on reducing the magnitudes of the time-averaged pressure changes (related to the effects of the piston weight, the clearance-fluid damping and the outflow pressure losses).

References


[9] Padden H 2003 Development of a 0.2% high-speed dry piston prover Measurement Science Conf. (Anaheim, CA)


