Title: UNCERTAINTY ANALYSIS OF GAS FLOW MEASUREMENTS USING CLEARANCE-SEALED PISTON PROVERS IN THE RANGE FROM 0.0012 G/MIN TO 60 G/MIN

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ABSTRACT:
This paper deals with an uncertainty analysis of gas flow measurements using a compact, high-speed, clearance-sealed realization of a piston prover. A detailed methodology for the uncertainty analysis, covering the components due to the gas density, dimensional and time measurements, the leakage flow, the density correction factor and the repeatability, is presented. The paper also deals with the selection of the isothermal and adiabatic measurement models, the treatment of the leakage flow and discusses the need for averaging multiple consecutive readings of the piston prover. The analysis is prepared for the flow range (50000:1) covered by the three interchangeable flow cells. The results show that using the adiabatic measurement model and averaging the multiple readings, the estimated expanded measurement uncertainty of the gas mass flow rate is less than 0.15 % in the flow range above 0.012 g/min, whereas it increases for lower mass flow rates due to the leakage flow related effects. At the upper end of the measuring range, using the adiabatic instead of the isothermal measurement model, as well as averaging multiple readings, proves important.

KEYWORDS:
piston prover, uncertainty analysis, adiabatic measurement model, leakage flow, CMC
1 INTRODUCTION

The piston-prover concept is widely used for primary standards in the field of gas flow measurements [1–10]. The general principle of operation is based on determining the time interval that a piston needs to pass a known volume of gas at a defined pressure and temperature. A general model for the mass flow rate can be expressed with:

\[ q_m = \frac{V_m}{\Delta t} + \left( \rho_{m,2} - \rho_{m,1} \right) \frac{V_d}{\Delta t} + q_{m,l}, \]  

where \( V_m \) is the measuring volume of the gas collected by the piston prover during the interval \( \Delta t = t_2 - t_1 \), \( \rho_{m,2} \) is the mean density of the gas in the measuring volume at the time \( t_2 \), \( V_d \) is the connecting volume of the gas between the meter under test and the piston at the time \( t_1 \), \( (\rho_{d,2} - \rho_{d,1}) \) is the change in the mean density of the gas in the connecting volume during the \( \Delta t \) interval, and \( q_{m,l} \) is the leakage mass flow rate.

This article deals with a commercially available, high-speed, clearance-sealed realization of the piston prover [11,12] that is schematically shown in Fig. 1. Such a prover is used in our accredited calibration laboratory as a primary standard for gas flow rate measurements and it consists of a base and three interchangeable flow cells with overlapping measuring ranges from 0.0012 g/min to 60 g/min (from 0.001 l/min to 50 l/min) for air flow at ambient conditions.

The base contains the processor, the time base and the atmospheric pressure sensor, while each of the flow cells houses a graphite piston and glass cylinder assembly with integrated temperature and pressure gauge sensors.
In our earlier work [13,14] we identified some deficiencies of the isothermal measurement model employed by the manufacturer in the piston prover and then proposed and developed an improved adiabatic measurement model. The adiabatic model was further amended [15,16] with the compensation of the heat exchange related effects that arise because of the temperature difference between the gas flow and the cylinder wall. In both stages the adiabatic measurement model was also experimentally validated. In [17] we presented an uncertainty analysis of the mass flow rate measured using the flow cell C of the piston prover.

Fig. 1. Scheme of the clearance-sealed piston prover.
The aim of this article is to present a more detailed methodology of the uncertainty analysis for the mass flow rate measurements using a clearance-sealed piston prover under ambient conditions that includes and extends our previous findings. The analysis also deals with the selection of the measurements model and discusses the need for averaging multiple consecutive readings of the piston prover. The presented uncertainty analysis is made for the flow range covered by all three flow cells (50000:1). We show that, with an appropriate treatment of the leakage flow, the nominal lower end value of 0.006 g/min for the smallest flow cell, declared by the manufacturer, can be decreased by five times, down to 0.0012 g/min. The evaluation of the uncertainties is made according to JCGM 100: 2008 [18].

The paper is organized as follows. Section 2 of the article outlines the applied measurement models, while Section 3 describes the measurement system. The individual contributions to the uncertainty of the mass flow rate due to the gas density, the dimensional and time measurements, the leakage flow rate, the density correction factor and the repeatability of the measured mass flow rate are discussed in Section 4. The final results of the analysis are given in Section 5 and include comparisons of the resulting uncertainties of the mass flow rate of dry air for isothermal and adiabatic measurement models and also for single or multiple averaged readings from the piston prover.
2 MEASUREMENT MODEL

The measurement model employed in the piston prover under discussion can be written as:

$$q_m = \rho(P_a, T) \left( \frac{V_m^*}{\Delta t} + q_{v,l}^{(p)} \right) \varepsilon_\rho .$$

(2)

The nominal gas density $\rho(P_a, T)$ at the atmospheric pressure $P_a$ and the time-averaged gas temperature in the piston prover $T$ are calculated using the equation for a real gas:

$$\rho(P_a, T) = \frac{P_a}{Z(R/M)T} ,$$

(3)

where $Z$ is the compressibility factor, which is pressure and temperature dependent, $R$ is the universal gas constant and $M$ is the molar mass of the gas. The effective measuring volume $V_m^*$ is expressed as

$$V_m^* = L_m \frac{\pi(D+\delta)^2}{4} ,$$

(4)

where $L_m$ is the distance passed by the piston in the time interval $\Delta t$, $D$ is the piston diameter, $D + \delta$ is the effective diameter of the cylinder, where $\delta$ is the clearance thickness, and $q_{v,l}^{(p)}$ is the Poiseuille leakage flow rate. The cylinder diameter is reduced from $D + 2\delta$ to $D + \delta$ in order to account for the Couette leakage flow component, which arises due to the piston movement relative to the cylinder wall (in a small clearance the mean gas velocity is nearly half of the piston velocity [19]). The density correction factor $\varepsilon_\rho$ accounts for the variations in the density of the gas relative to $\rho(P_a, T)$. Considering the improved adiabatic model that accounts for the quasi-adiabatic nature of the relatively high-frequency oscillations of the gas, the correction factor reads as [13,14]:
\[ \varepsilon_p^{(A)} = 1 + \frac{p_{12}}{P_a} + \frac{1}{\gamma} \left( \frac{p_2 - \overline{p}_{12}}{P_a} + \frac{p_2 - p_1}{P_a} \frac{V_d}{V_m^*} \right). \quad (5) \]

where \( p_1 \) and \( p_2 \) are the gauge pressures at the times \( t_1 \) and \( t_2 \), respectively, \( \overline{p}_{12} \) is the time-averaged value of the gauge pressure during the timing cycle and \( \gamma \) is the adiabatic index. In the uncertainty analysis, the adiabatic model will also be compared to the isothermal measurement model:

\[ \varepsilon_p^{(T)} = 1 + \frac{p_2}{P_a} + \frac{p_2 - p_1}{P_a} \frac{V_d}{V_m^*}, \quad (6) \]

which is originally employed in the piston prover.

The measurement model (2-5) can be derived from the general model (1) using the following assumptions: a spatially homogenous gas density in \( V_m \) and \( V_d \) for each time instant, \( \rho_1 = \rho_{d,1} \) and \( \rho_2 = \rho_{m,2} = \rho_{d,2} \), a thermal equilibrium between the inlet gas flow and the cylinder wall, and negligible gas compressibility effects in the density correction factor, \( Z_1 = Z_2 = Z \).

Accounting for the heat exchange effects, which arise due to the temperature difference between the gas flow and the cylinder wall, the density correction factor is modified to [16]:

\[ \varepsilon_p = \varepsilon_p^* \left( 1 - k_r (T - T_{wall}) \right), \quad (7) \]

where \( \varepsilon_p^* \) is the adiabatic or isothermal correction factor given by equations (5) or (6), \( k_r \) is the sensitivity coefficient, the value of which depends on the selected flow cell as well as on the measured flow rate, and \( T_{wall} \) is the temperature of the cylinder wall.
3 MEASUREMENT SYSTEM

The mass flow rate of air is measured by the piston prover that consists of a base (Sierra Instruments, Cal=Trak SL-800) and three interchangeable flow cells with overlapping flow ranges: cell A (SL-800-10, flow range: 0.0012 g/min – 0.6 g/min, $D = 0.93$ cm, $L_m = 10.17$ cm), cell B (SL-800-24, flow range: 0.06 g/min – 6 g/min, $D = 2.40$ cm, $L_m = 10.17$ cm) and cell C (SL-800-44, flow range: 0.6 g/min – 60 g/min, $D = 4.44$ cm, $L_m = 7.61$ cm).

In the piston prover the measurement time interval $\Delta t$ is measured using the internal time base. The piston diameter $D$ and the distance $L_m$ that a piston passes between two optical sensors are obtained from the external dimensional calibrations. The clearance thickness $\delta$ is evaluated from the analytical model for the Poiseuille leakage flow rate [11]. The Poiseuille leakage flow rate $q_v^{(p)}$ is determined periodically by the external calibration provider as well as using the in-house calibrations. In the second case, we are using the validated dynamic summation method, which is described in Appendix A. The values of the internal connecting volume $V_d$ for each flow cell, which includes the interior passages, the valve and the portion of the cylinder below the point at which the timing begins, are provided by the manufacturer.

The pressure $P_a$ is measured using an atmospheric pressure sensor located in the prover’s base and the temperature $T$ is measured using the integrated temperature sensor at the inlet of the cylinder for each flow cell. The density $\rho$ of the selected gas, which depends on the absolute pressure and the temperature, is obtained from the REFPROP database [20].

The characteristic gauge pressures $p_1$ and $p_2$ are measured using the internal pressure sensor of the flow cell. In order to apply the adiabatic correction model, instead of the isothermal used by
the manufacturer, the time-averaged pressure $\bar{p}_{12}$ is measured by adding an external pressure sensor (Validyne P855, measuring range: $-1.4$ kPa to $1.4$ kPa, voltage output: $-5$ V to $5$ V, low pass filter at $250$ Hz / $-3$ dB). This external sensor is connected in parallel with the internal pressure gauge sensor and its voltage output is measured with a DAQ board (National Instruments, USB-6251 BNC). The communication with the piston prover and with the DAQ board, as well as the processing of the measurement signals, is realized using LabVIEW software (National Instruments, Ver. 10.0).

The piston prover is connected to a stable flow source, which is realized using critical nozzles (TetraTec Instruments, array of five Venturi-shaped critical nozzles, flow range: $0.6$ g/min – $60$ g/min) or with a set of mass flow controllers (Bronkhorst F-201CV, four controllers, maximum flow rates: $0.013$ g/min (2 pieces), $0.13$ g/min, $1.3$ g/min, control stability: $\leq 0.1$ % max. flow rate). The smallest, but still suitable, mass flow controller is selected outside the flow range of the critical nozzle array.

The measurement result is obtained as a single reading or as the moving-average value of multiple consecutive readings. With a single reading, we mean a value obtained from a single measurement cycle (run) of the piston prover. The moving-average value is, in general, obtained by averaging ten consecutive readings, except for the smallest flow rates of the flow cell A ($q_m \leq 0.006$ g/min), where only three readings are averaged, because of the relatively long time (about $10$ min at $0.0012$ g/min) needed to carry out a single measurement.
4 EVALUATION OF THE UNCERTAINTY CONTRIBUTIONS

4.1. Nominal gas density

The components of the standard relative uncertainty of the air density are presented in Table 1. The uncertainties of the absolute pressure and the temperature are determined using the calibration results and the calibration history (time drift) of the sensors. The stated uncertainty of the temperature is equal for all three flow cells. The contribution of the compressibility factor represents the uncertainty of the underlying REFPROP models and is estimated by a comparison of its values with those of a particular reference model (e.g., CIPM-2007 formula for air [21]). The molar mass is affected by the actual composition of the gas applied in the measurement process. The dry air used with the measurement system in our laboratory is obtained by passing compressed air through a set of filters and the desiccant dryer (Kaeser DC 7.5, dew point = –70°C). Following Ref. [21], we can calculate the uncertainty of the molar mass of dry air by combining the uncertainties of the mole fraction of CO₂ (x_{CO₂}) and of the relative humidity of the air (ϕ):

\[
\frac{u(M)}{M} \approx \sqrt{\left(0.41 u(x_{CO₂})\right)^2 + \left(0.01 u(\phi) \frac{100 \text{ kPa}}{p_\phi}\right)^2},
\]

where \(p_\phi\) is the pressure in kPa at which the relative humidity is measured. The mole fraction of CO₂ is not measured, but it is assumed to be equal to \(4 \times 10^{-4}\) and to be encompassed by a rectangular distribution having a half-width of \(4 \times 10^{-4}\), which leads to a standard uncertainty of \(u(x_{CO₂}) = 4 \times 10^{-4} / \sqrt{3}\). The relative standard humidity is measured with a capacitive sensor at 500 kPa, and its relative standard uncertainty is equal to 5%.
Table 1. Components of the air density standard uncertainty, \( u(\rho) / \rho = \sqrt{\sum (c_{\text{rel},i} u(x_i)/x_i)^2} \)

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( c_{\text{rel},\xi} u(x_\xi) / x_\xi )</th>
<th>( c_{\text{rel},\xi} u(x_\xi) / x_\xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a )</td>
<td>( u(P_a) / P_a )</td>
<td>0.026 %</td>
</tr>
<tr>
<td>( T )</td>
<td>( -u(T) / T )</td>
<td>-0.026 %</td>
</tr>
<tr>
<td>( Z )</td>
<td>( -u(Z) / Z )</td>
<td>-0.010 %</td>
</tr>
<tr>
<td>( M )</td>
<td>( u(M) / M )</td>
<td>0.014 %</td>
</tr>
<tr>
<td>( u(\rho) / \rho )</td>
<td>0.040 %</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2. Dimensional and time measurements

The term \( V^*/\Delta t \) depends on the dimensional quantities of the piston and the cylinder \( (D, L_m, \delta) \) as well as the measurement time interval \( \Delta t \). The contributions to the relative standard uncertainty of the uncorrected volume flow rate are shown in Table 2 (\( \delta \ll D \) can be considered in the notation of the sensitivity coefficients, because \( \delta \) is smaller than \( 10^{-3} \) cm for all flow cells). The uncertainties of the respective quantities are determined using the calibration results and the calibration history (time drifts), while the uncertainty, \( u(\delta) \), is estimated according to Ref. [11].

Before a dimensional calibration of the piston diameter \( D \) is performed, the flow cell is disassembled and the piston-cylinder assembly is cleaned, with the intention being to remove any dirt or dust that could obstruct the smooth motion of the piston. It is very likely that the piston cleaning slightly affects its diameter, so the actual drift of the diameter is probably smaller than the value considered in the analysis. The distance \( L_m \) that a piston passes between two optical sensors is also determined after the flow cell is reassembled, meaning that the difference between the values obtained in the last two calibrations represents not only the time
drift (attributed to, e.g., the ageing of the optical sensors) but also to the reproducibility of the optical sensors’ reassembly. Therefore, the present analysis estimates the time drift of \( L_m \) as one half of the difference between the values obtained in the last two calibrations.

Table 2. Relative components of the uncorrected volume flow uncertainty for all three flow cells, \( u(V_m^+/\Delta t)/(V_m^+ / \Delta t) = \sqrt{\sum (c_{rel,i} u(x_i)/x_i)^2} \)

<table>
<thead>
<tr>
<th>( \xi_i )</th>
<th>( c_{rel,i} u(x_i)/x_i )</th>
<th>Cell A</th>
<th>Cell B</th>
<th>Cell C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta t )</td>
<td>( -u(\Delta t)/\Delta t )</td>
<td>( -0.018 % )</td>
<td>( -0.018 % )</td>
<td>( -0.018 % )</td>
</tr>
<tr>
<td>( L_m )</td>
<td>( u(L_m)/L_m )</td>
<td>0.016 %</td>
<td>0.013 %</td>
<td>0.013 %</td>
</tr>
<tr>
<td>( D )</td>
<td>( 2u(D)/D )</td>
<td>0.026 %</td>
<td>0.0070 %</td>
<td>0.0070 %</td>
</tr>
<tr>
<td>( \delta )</td>
<td>( 2u(\delta)/D )</td>
<td>0.015 %</td>
<td>0.0065 %</td>
<td>0.0044 %</td>
</tr>
<tr>
<td>( u(V_m^+ / \Delta t)/(V_m^+ / \Delta t) )</td>
<td></td>
<td>0.038 %</td>
<td>0.024 %</td>
<td>0.023 %</td>
</tr>
</tbody>
</table>

4.3. Leakage flow rate

The leakage flow rates \( q_{vl}(^p) \) for all three flow cells equal to: 0.122 cm\(^3\)/min (cell A), 0.191 cm\(^3\)/min (cell B) and 0.78 cm\(^3\)/min (cell C) having standard uncertainties \( u(q_{vl}(^p)) \) of: 0.004 cm\(^3\)/min (cell A), 0.019 cm\(^3\)/min (cell B) and 0.13 cm\(^3\)/min (cell C). The uncertainties are given by their time drift and the repeatability, which is estimated by the experimental standard deviation of consecutive measurements of the leakage flow rate. For the flow cell A the time drift (0.018 cm\(^3\)/min) alone contributes about 2 \% to the standard uncertainty of the
measured gas flow rate at its lower end of the measuring range. Therefore a leakage flow rate is determined immediately before or after any of the performed measurements or calibrations and the uncertainty of the leakage volume flow rate for flow cell A is estimated by accounting only for its repeatability.

The described approach probably overpredicts the actual uncertainty related to the leakage flow rate in all cases, since its estimated repeatability is also partly included in the estimated repeatability of the measured mass flow rate (see Section 4.5). The dispersion of the leakage flow rate is probably related to the variations of the piston position relative to the cylinder walls and to the piston rocking [11].

4.4. Density correction factor

A detailed uncertainty analysis of the density correction factors for the adiabatic and isothermal models was already presented in [14]. Here, the analysis is amended with an additional uncertainty component \( u(\Delta T) \), which represents the heat exchange effects arising due to the temperature difference between the temperatures of the inlet gas flow and the cylinder wall (see [16] for more details). In the current analysis, this contribution accounts for the fact that the heat-transfer correction equation (7) is not applied for the calculation of the density correction factor. Table 3 lists the significant components of the standard uncertainty for the adiabatic model (considering \( \varepsilon_p \approx 1 \) in the notation of the sensitivity coefficients).
Table 3. Components of the density correction factor uncertainty,
\[ u(\varepsilon_p) / \varepsilon_p = \sqrt{\sum_i \left( c_{rel,i} u(x_i) / x_i \right)^2} \]

<table>
<thead>
<tr>
<th>(x_i)</th>
<th>(c_{rel,i} u(x_i) / x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_1)</td>
<td>(- \frac{1}{\gamma} \frac{V_d}{V^*_m} \frac{p_1}{p_a} u(p_1))</td>
</tr>
<tr>
<td>(p_2)</td>
<td>(\frac{1}{\gamma} \left(1 + \frac{V_d}{V^*_m}\right) \frac{p_2}{p_a} u(p_2))</td>
</tr>
<tr>
<td>(\overline{p}_{12})</td>
<td>(\frac{\gamma - 1}{\gamma} \frac{\overline{p}<em>{12}}{\gamma} u(\overline{p}</em>{12}))</td>
</tr>
<tr>
<td>(V_d)</td>
<td>(\frac{p_2 - p_1}{\gamma} \frac{V_d}{V^*_m} \frac{u(V_d)}{p_a})</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(- \frac{1}{\gamma} \left( \frac{p_2 - \overline{p}_{12}}{p_a} + \frac{p_2 - p_1}{\gamma} \frac{V_d}{V^*_m} \right) \frac{u(\gamma)}{\gamma})</td>
</tr>
<tr>
<td>(\Delta T)</td>
<td>(u(\varepsilon_p^{\Delta T}))</td>
</tr>
</tbody>
</table>

The estimated standard uncertainty of the gauge pressures \(p_1\) and \(p_2\) is equal to 5 Pa (for all three flow cells) and the estimated standard uncertainty of \(\overline{p}_{12}\) is equal to 2 Pa. The characteristic pressures were measured for each flow cell for at least seven different flow rates distributed across its measuring range with fifty consecutive readings taken at each flow rate (fifteen for the lowest two flow rates of flow cell A). Fig. 2 shows the resulting second-order approximations of the pressures \(p_1\), \(p_2\) and \(\overline{p}_{12}\) for all three flow cells, which enabled continuous representation of flow rate dependent uncertainties in the subsequent analysis. It can be seen that the differences between particular pressures become significant only for the flow cell C, which is related to the more intense pressure oscillations during the prover’s timing cycle.
Fig. 2. The approximations of characteristic pressures in the timing cycles of the piston prover for all three flow cells (△ cell A, ○ cell B, □ cell C)

The internal connection volumes $V_d$ for a particular flow cell equal: $1.31 \cdot V_m^*$ (cell A), $1.28 \cdot V_m^*$ (cell B) and $1.69 \cdot V_m^*$ (cell C). The value of the standard uncertainty is equal to 2.46 cm$^3$ for flow cell A and 6.29 cm$^3$ for flow cells B and C. The estimated values cover the possible change of the connecting volume, which corresponds to the volume of a 0.5 m long connecting tube that is typically used with the individual flow cell. The adiabatic index $\gamma$ for dry air is equal to 1.4 and is encompassed by a rectangular distribution having a half-width of 0.1 that leads to a standard uncertainty of $u(\gamma) = 0.1/\sqrt{3}$ [14].

In Ref. [16] the heat transfer in the cylinder was studied theoretically and experimentally. Supported by these findings and the results of additional simulations carried out for the smallest flow cells it was established that the relative heat-transfer correction (7) does not
exceed 0.035% if the maximum temperature difference between the inlet flow and the cylinder wall is 0.15 K below 6 g/min or up to 0.5 K at 60 g/min. It was proven that the required temperature difference can be assured by the proper temperature stabilization of the piston prover and its surroundings. So, the standard uncertainty estimation of \( u(\varepsilon_\Delta T) = 0.035\% / \sqrt{3} \) can be reasonably attributed to the heat exchange related effects.

Fig. 3 shows the individual contributions and the resulting relative uncertainty of the density correction factor using the adiabatic model for flow cell C. The increase at higher flow rates is related to the increased difference between the characteristic pressures, which is reflected in the increased uncertainty contributions of \( \gamma \) and \( V_d \). The largest uncertainty contribution in the entire measuring range is related to the heat exchange effects in the cylinder. For the other two flow cells, A and B, the value of the relative uncertainty of the density correction factor is smaller than 0.023% across their entire measuring range. The temperature inhomogeneity also remains as the largest uncertainty contribution in these two cells.
Fig. 3. Contributions to the uncertainty of the density correction factor for flow cell C

In Fig. 4, the values of the adiabatic (5) and isothermal (6) density correction factors are compared for flow cell C. The values of the correction factors for both models are close to 1 across the entire measuring range of the piston prover, reaching the highest values of approximately 1.006 at the highest flow rates. Next, we define the difference between the isothermal and adiabatic models: $\delta_\varepsilon = \left| \frac{\varepsilon^{(T)} - \varepsilon^{(A)}}{\varepsilon^{(A)}} \right|$. It was established that $\delta_\varepsilon$ is lower or equal to 0.01 % for the flow cells A and B because of the relatively small differences between the characteristic pressures. On the other hand, as shown in Fig. 4, the $\delta_\varepsilon$ in flow cell C increases to above 0.05 % for flow rates greater than 10 g/min (up to 0.13 % at 40 g/min), which is in the range of the increased pressure oscillations in the measuring cycle.
Fig. 4. Values of the adiabatic and isothermal density correction factor $\varepsilon_\rho$ and their relative difference $\delta_\varepsilon$ for flow cell C

### 4.5. Repeatability

The repeatability was estimated from the same set of measurements that were conducted to determine the characteristic pressures in the timing cycle of the piston prover (see Section 4.4). In such manner fifty (fifteen) single readings and forty-one (thirteen) moving-average readings at each flow rate were obtained (numbers in parentheses refer to the lowest two flow rates of flow cell A). Fig. 5 presents the repeatability estimated as the experimental standard deviation, $s(q_m)$, of: (i) single readings, (ii) moving-average readings. The results show that averaging consecutive readings of the piston prover decreases the experimental standard deviation on average by almost a factor of 3. The worse repeatability at higher flow rates, especially evident when only a single reading is taken into account, is related to the increased pressure
oscillations and to the quantization error of the measurement time interval. Fig. 5 also shows the repeatability envelopes considered further in the uncertainty analysis.

Fig. 5. The relative experimental standard deviation for single and moving-average readings

(△ cell A, ○ cell B, □ cell C)
COMBINED AND EXPANDED UNCERTAINTY

The combined standard uncertainty of the mass flow rate can be expressed by combining the individual components presented in the previous section. The components and their sensitivity coefficients are listed in Table 4 (considering $q_{i,j}^{(p)} \ll V_m^* / \Delta t$ in the notation of the sensitivity coefficients).

Table 4. Components of the combined standard mass flow rate uncertainty,

\[
u(q_m) / q_m = \sqrt{\sum \left( c_{rel,j} u(x_j) / x_j \right)^2} \]

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$c_{rel,j} u(x_j) / x_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$u(\rho) / \rho$</td>
</tr>
<tr>
<td>$V_m^* / \Delta t$</td>
<td>$u(V_m^* / \Delta t) / (V_m^* / \Delta t)$</td>
</tr>
<tr>
<td>$\varepsilon_{\rho}$</td>
<td>$u(\varepsilon_{\rho}) / \varepsilon_{\rho}$</td>
</tr>
<tr>
<td>$q_{i,j}^{(p)}$</td>
<td>$u(q_{i,j}^{(p)}) / (V_m^* / \Delta t)$</td>
</tr>
<tr>
<td>repeatability</td>
<td>$s(q_m) / q_m$</td>
</tr>
</tbody>
</table>

Fig. 6 shows the combined standard relative uncertainty of the mass flow rate for the flow cell A when using the moving-average readings and the adiabatic measurement model. The contributions of particular components listed in Table 4 are also presented in the graph. In the considered case the combined standard uncertainty is close to 0.5 % for the smallest flow rate at 0.0012 g/min, from where it decreases to about 0.07 % at 0.015 g/min and remains below this value up to the maximum flow rate of 0.6 g/min. The increase of the uncertainty below 0.015 g/min is mostly attributed to the leakage flow component and to the repeatability of the
measured mass flow rate. At higher flow rates the most important uncertainty contributions are
the uncertainties of the uncorrected measuring volume flow rate, the density of the air and the
density correction factor (the first two contributions are almost identical and hardly
distinguishable in the graph). The last three contributions remain the most important also for
the larger two flow cells B and C. When using the moving-average readings the repeatability
contribution for these two cells remains relatively small (0.01 %) and the combined relative
standard uncertainty remains smaller than 0.07 % across their entire measuring.

Fig. 6. Combined mass flow rate standard relative uncertainty for the moving-average readings
of flow cell A, including the contributions of the individual components

Finally, the expanded measurement uncertainty of the mass flow rate measured by the piston
prover is evaluated as \( U(q_m) / q_m = k \cdot u(q_m) / q_m \) using the coverage factor, \( k \), for 95.45 %
confidence interval by taking into account the effective degrees of freedom, which are calculated according to the Welch-Satterthwaite formula [18]. In the case of the isothermal model, the expanded relative uncertainty was evaluated by treating the relative difference between the isothermal and the adiabatic density correction factor, $\delta_c$, as the uncorrected systematic error [14]: $U(q_m) / q_m = k \cdot u(q_m) / q_m + \delta_c$. In Fig. 7 the expanded relative uncertainty of the air mass flow rate is presented for the measuring range of all three flow cells using three different measurement methods.

Fig. 7. The relative expanded uncertainty for different measurement methods (△ cell A, ○ cell B, □ cell C)
At flow rates less than 0.015 g/min the expanded uncertainty increases steeply to approximately 1 % because of the leakage flow contribution. The selection of the adiabatic or the isothermal correction model is not very influential for flow cells A and B (observed differences only up to 0.01 %), but it considerably affects the measurements in the flow range covered by flow cell C. If the isothermal model is applied, the expanded uncertainty increases steeply for flow rates above 3 g/min and reaches its peak of 0.24 % at about 40 g/min, whereas for the adiabatic model the uncertainty remains below 0.11 %.

The difference between the single readings and the moving-average readings is more evident at the lower (< 0.06 g/min) and the upper (> 20 g/min) ends of the measuring range, where the values of $U(q_m)/q_m$ differ up to as much as about 0.15 % and 0.06 %, respectively. The observed difference is related to the increased non-repeatability of the single-reading measurements (see Fig. 5) and the increase of the coverage factor for 95% confidence interval at both ends of the flow range (up to $k = 2.4$ at 60 g/min).
6 CONCLUSIONS

A detailed methodology for the evaluation of the measurement uncertainty of a clearance-sealed piston prover is presented. The piston prover under consideration uses three interchangeable flow cells to cover the mass flow range between 0.0012 g/min and 60 g/min. The uncertainty analysis accounts for the contributions due to the gas density, the dimensional and time quantities, the leakage flow rate, the density correction factor and the repeatability of the measured mass flow rate.

The uncertainty analysis, which was made for flow measurements of dry air, identifies the largest individual contributions to the combined measurement uncertainty of the mass flow rate. Uncertainties related to the air density, the dimensional and the time quantities as well as the density correction factor are approximately of the same order of magnitude across the entire mass flow range. Therefore, reducing only one of them would not considerably affect the resulting combined uncertainty. Exceptions are the lower and upper ends of the observed measuring range, where the proper treatment of the leakage flow effects and the repeatability effects, respectively, can decrease the combined uncertainty. The former requires the leakage flow rate to be determined immediately before or after the measurements and the latter requires a sufficient number of averaged readings. In the range of increased pressure oscillations, \( q_m > 10 \text{ g/min} \), the conducted measurements show that averaging multiple readings instead of taking only a single reading as the measurement result can reduce the repeatability of the measured mass flow rate by a factor of 3 or 4.

The paper also demonstrates that for mass flow rates greater than about 10 g/min the selection of the density correction model proves to be very important. Using the developed adiabatic model instead of the isothermal one, which is originally applied in the piston prover, can
reduce the combined uncertainty by up to 50%. However, in the studied configuration of the piston provers, its use requires an external fast response pressure gauge sensor to be fitted in parallel with the internal gauge sensor of the flow cell.

Using the adiabatic measurement model, including the averaging of multiple readings, the estimated expanded measurement uncertainty of the gas mass flow rate for the clearance-sealed prover is smaller than 0.15% in the flow range between 0.012 g/min and 60 g/min. It was demonstrated that with a proper treatment of the leakage flow effects, the measuring range of the smallest flow cell was extended down to 0.0012 g/min. At this flow rate the expanded uncertainty increases up to almost 1%, which is mostly because of the repeatability of the leakage flow rate during the measurement.

The analysis also suggests that the dimensional calibration procedure for the piston diameter and the piston travel distance should be modified in order to provide the relevant history data for the estimation of the time drifts of both quantities. It would also be advisable to perform additional dimensional calibrations of the cylinder inner diameters that would enable direct calculations of the clearance thicknesses, thereby removing the need to use an analytical model for its estimation.
APPENDIX A.: DYNAMIC SUMMATION METHOD

The method is used to measure the leakage flow rate $q_{l,j}^{(p)}$ in the clearance-sealed piston prover. As shown in Fig. A.1, the gas is supplied from two stable flow sources to two parallel flow branches, each restricted by a valve, which reunite before the inlet to the piston prover. During the measurement the uncorrected readings of the piston prover have to be recorded, which can be achieved by setting the leakage volume flow rate in equation (2) to zero. So, the actual mass flow rate ($q_m$) can be written as the sum of the uncorrected reading of the piston prover ($q_m^*$) and the leakage mass flow rate past the piston cylinder clearance ($q_{m,l}$):

$$q_m = q_m^* + q_{m,l}. \quad (A.1)$$

The mass flow rate is consecutively measured from each flow source separately ($q_{m1}, q_{m2}$) by closing the valve in the other branch, as well as from both flow sources simultaneously ($q_{m1+m2}$). By closing a valve in a particular branch the gas is diverted to the ambient environment (shown with dashed lines in Fig. A.1). Assuming that all mass flow rate sources remain stable during the measurement, the following holds true:

$$q_{m1+m2} + q_{m,l} = q_{m1}^* + q_{m,l}^* + q_{m2}^* + q_{m,l}^*. \quad (A.2)$$

Hence, it follows that the leakage mass flow rate can be determined by subtracting the two uncorrected flow readings obtained in the separate measurements for each flow source from the uncorrected reading obtained in the simultaneous measurement:

$$q_{m,l} = q_{m1+m2}^* - q_{m1}^* - q_{m2}^*. \quad (A.3)$$

Finally, the leakage volume flow rate as defined in the measurement model of the piston prover is given by

$$q_{l,j}^{(p)} = \frac{q_{m,l}}{p_\varepsilon_p}, \quad (A.4)$$
where \( \rho \) and \( \varepsilon_p \) are taken as the average values during the measurement.

Figure A.1. Schematic representation of the experimental setup
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